Technical Notes

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Prediction of Outer Layer Mixing Lengths in Turbulent Boundary Layers

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IN a recent issue of the *AIAA Journal*,[1] Pletcher presented a method for determining the mixing length of a two-layer eddy viscosity model for a turbulent boundary layer at low Reynolds numbers which permits "... the accurate prediction of low Reynolds number flows without recourse to model alterations in the form of Reynolds number functions." Incorporation of the model into a finite-difference calculation method was shown to accurately predict low Reynolds number skin friction in supersonic flow, but a comparison with available mixing length scales was not made. The purpose of this Note is to compare mixing length predictions by Pletcher's method with some recent values derived from turbulent boundary-layer profiles in Refs. 2 and 3.

The two-layer eddy viscosity model can be formulated as follows:

$$\epsilon = \rho \ell^2 \mathrm{d}u/\mathrm{d}y \tag{1}$$

where

$$\ell = kyD \quad \text{in the wall region} \tag{2}$$

$$\ell = C\delta \quad \text{in the outer region} \tag{3}$$

For $\mathrm{d}p/\mathrm{d}x = 0$, adiabatic flows, the slope of the mixing length near the wall, k, is usually taken to be about 0.4. The wall damping factor is $D = 1-\exp(-y^+/26)$ from Ref. 4. The value of C, until recently, has been assumed constant with a value between 0.08 and 0.09; however, it is now common practice to let C increase with decreasing Reynolds number by means of an auxiliary equation (Refs. 5 and 6). The increase in the outer layer mixing length at low Reynolds numbers applies only to flat plate boundary layers.[3] For nozzle walls the outer mixing length can actually decrease at low values of δ^+ ($\delta^+ = \delta u_\tau/\nu_w$); however, in both cases the same value is approached at high Reynolds number.

As usually applied, the mixing length from Eqs. (2) and (3) is determined by using Eq. (2) until $\ell/\delta = C$. At this point Eq. (3) holds until the edge of the boundary layer is reached. As shown in Fig. 1, for values of δ^+ below about 500, there will be significant damping at the point of switchover between Eqs. (2) and (3). Pletcher assumes: 1) the form of D and the value of k are independent of the Reynolds number, and 2) "that the mixing length will closely approach the fully turbulent value prior to becoming independent of distance from the wall, that is, no wake-like region will form until a fully turbulent region exists." The term "fully turbulent" as used in Ref. 1 denotes the logarithmic portion of the boundary-layer profile, i.e., that part described by $\ell = ky$. Pletcher evidently assumes that the flow is fully turbulent when wall damping is small ($D \simeq 1$). Assumption 2 is applied by allowing Eq. 2 to hold until a certain limiting value, y_L^+, is reached. In Ref. 1, $y_L^+ = 50$ was assumed. If, at $y_L^+ = 50$, ℓ/δ is greater than C in Eq. (3), then the larger value is retained until y/δ reaches 1. If ℓ/δ is less than C at $y_L^+ = 50$, the mixing length distribution from Eq. 2 continues until C is reached, and the division between the inner and outer regions occurs at $y/\delta = C/(kD)$. Mixing length distributions calculated using Pletcher's model are shown in Fig. 2. Note that if $\delta^+ < y_L^+$, damping will occur over the complete boundary layer.

Pletcher's use of $y_L^+ = 50$ as the point where wall damping will cease is admittedly arbitrary,[1] and this value determines the maximum value of ℓ/δ in the outer region according to the equation

$$(\ell/\delta)_{\mathrm{MAX}} = kD_L y_L^+/\delta^+ \tag{4}$$

where

$$D_L = D \text{ at } y_L^+$$

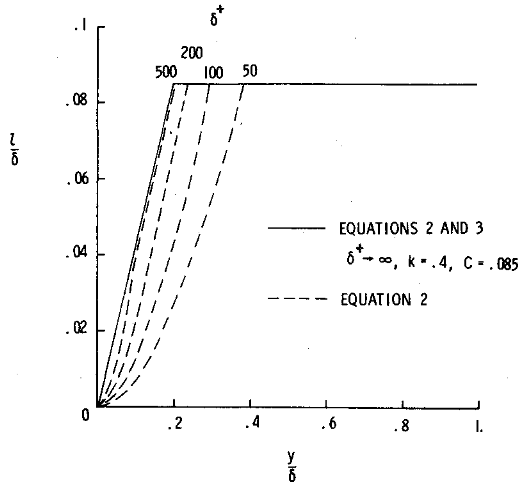

Fig. 1 Conventional mixing length model with wall damping.

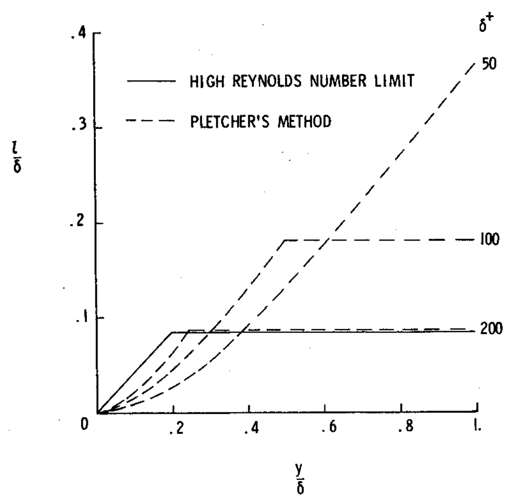

Fig. 2 Mixing length according to Pletcher.

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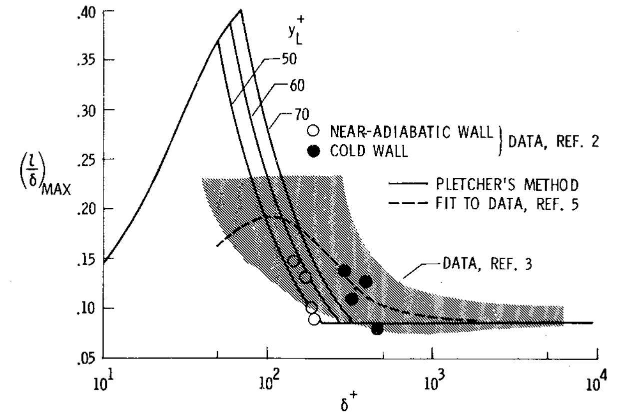

Fig. 3 Variation of outer mixing length with Reynolds number.

Figure 3 shows experimental values of $(\ell/\delta)_{MAX}$ for two-dimensional turbulent boundary layers[2,3,5] compared with calculations from Eq. (4) for $k=0.43$ (shown in Ref. 2 to fit the hypersonic helium data) and y_L^+ values of 50, 60, and 70. It can be seen that for $\delta^+ > y_L^+$ Pletcher's model predicts an increase in mixing length as δ^+ decreases, in agreement with the trend of the data. More extensive data may show that y_L^+ is a function of T_w/T_t, as implied by the data; however, the available experimental data are too sparse to say whether or not this is true.

References

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Detection of Boundary-Layer Transition Using Holography

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THIS Note presents some preliminary findings on the application of holography to the measurement of boundary-layer transition on a sharp tip, axisymmetric cone at zero angle of attack in a hypersonic, high Reynold's number flow. A pulse ruby laser is used in a modified, off axis, single pass schlieren system to record the holograms (Refs. 1-4), and both holographic schlieren and dual plate holographic interferometry are used to observe the transition phenomenon.

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Figure 1 shows typical results obtained for tests conducted in the Air Force Flight Dynamics Laboratory Mach 6, high Reynolds number wind tunnel. The schlieren photograph reveals pronounced changes in the boundary-layer thickness along the upper surface of the cone, and the circled region contains what the author believes is a turbulent burst. The accompanying interferogram is constructed from the same hologram of this flow so that direct correlation can be made between changes in the fringe pattern with changes visible in the schlieren photograph. Point-by-point comparisons show that as the boundary-layer thickness increases, the fringes appear to pull away from the wall and that the angles between the fringes and the surface of the cone increase. Since the density gradient is steeper and the boundary-layer thickness is greater for a turbulent boundary layer than for a laminar boundary layer, light waves traversing a turbulent boundary layer experience more refraction and greater phase shifts, and this means there are characteristic fringe patterns associated with each state of the boundary layer. Hence, it is possible to perform a fringe-by-fringe survey of an interferogram to locate precise points where the boundary layer is changing states. Examples of these characteristic fringe patterns are observable in Fig. 1 particularly in the enlargement of the burst, and from a fringe-by-fringe analysis, another burst can be identified (indicated by the arrow on the interferogram) on the lower surface of the cone. The burst on the lower surface appears to be considerably smaller than the one in the enlargment and, as measured from the interferogram, the smaller burst is approximately 4.25 mm long and 0.64 mm thick. The laminar boundary-layer thickness on both sides of this burst measures 0.24 mm. Thus far, for the general conditions of Fig. 1, bursts have been observed at distances measured from the cone tip which correspond to Reynolds numbers of 4.5-9.0 million, and the formation of the fully turbulent boundary layer occurs consistently at Reynolds numbers of 8-9.5 million. These results agree with those of other researchers investigating boundary-layer transition on similar configurations in similar flow fields.[5]

For this investigation, the ruby laser is adjusted to emit a light pulse that is approximately 35 nanoseconds in duration. This pulse represents the period during which the boundary layer is recorded in a hologram, and by assuming that the frequency of the boundary-layer instabilities has a much greater period, each hologram is an instantaneous measurement of the flow. The validity of this assumption can be determined by comparing representative periods of the boundary-layer instabilities to the period of the laser pulse. As discussed by Reshotko (Ref. 6), the nondimensional frequency, $\beta\nu/U^2$, is the relevant frequency parameter to use when boundary-layer instabilities are considered. In this parameter, U and ν are, respectively, the velocity and kinematic viscosity evaluated at the edge of the boundary layer ($U=3345$ fps, $\nu=1.508\times10^{-4}$ f^{2}ps), and β represents the characteristic angular frequency of the disturbance spectrum ($\beta=2\pi f=2\pi/\tau$, where τ is the period). Precise values of β are not known for this investigation, but from the Mach 5.8 data of Demetriades,[7] representative values of the nondimensional frequency can be used in conjunction with U and ν to determine an approximate period for the boundary-layer instabilities. This period is in the range of 1.7-8.5 microseconds, which is approximately three orders of magnitude longer than the period of the laser pulse. Hence the assumption that the holograms contain instantaneous measurements of the boundary layer is reasonable.

The intent of this holographic application is to obtain new and unique boundary-layer measurements of the transition process, but difficulties arise when an attempt is made to define the structure of the burst solely from the optical data. From the results of Fig. 1, one can see that the boundary layer is not axially symmetric near or in a burst, and because multi-viewing is not feasible for the facility currently being used, correct orientation of the bursts in not known. Many different